Comment on "Magnus Force in Superfluids and Superconductors"

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The effective action for the Josephson junction arrays (JJA) should contain the topological term, which violates the particle-hole symmetry. This term is responsible for the nonzero Magnus force acting on the vortex. The Magnus force is however small because the parameter of the particle-hole asymmetry in superconductors is of order $\Delta^2/E_F^2 \ll 1$.

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Sonin[1] recently presented an extreme view on the Magnus force acting on vortices in the Josephson junction arrays (JJA). He suggested that the "Hall effect is *exactly* absent in the classical theory of JJA which neglects the charge quantization. Since the Hall effect is linear in the amplitude of the effective Magnus force, the latter also vanishes in the classical JJA. This statememnt directly follows from the symmetry of the dynamic equations." He argued that on the microscopical level the suggested symmetry of equations "is a direct result of the *particle-hole symmetry*". Here we discuss why this is not true.

The vortices in the arrays could be considered as massive particles with long-range Coulomb interaction[2,3]. In the experiment[4] the straightforward ballistic motion of vortices was observed which implies the Magnus force, acting on a vortex perpendicular to its velocity, is absent or is very small. This was also confirmed by more recent experiments: vortices move perpendicular to the driving current[5], and no Hall effect was detected in the system[6].

On the macroscopic level the general form of the balance of forces acting on the vortex in the case when the normal component is pinned by the crystal lattice or by JJA is the Eq.(23) of Ref.[1]:

$$\rho_M \mathbf{v}_L \times \vec{\kappa} + \eta \mathbf{v}_L = \mathbf{j} \times \vec{\kappa}. \tag{1}$$

Here the first term on the left-hand side of the equation is the definition of the *effective Magnus force* according to[1], $\vec{\kappa}$ is the circulation vector and $\mathbf{v}_L$ is the velocity of the vortex with respect to the array. The second term on the left-hand side is the friction force which we do not discuss. The force on the right-hand side is produced by the electric supercurrent $\mathbf{j}_e = (e/m)\mathbf{j}$, where $\mathbf{j} = \rho_s \mathbf{v}_s$ is the mass current.

In the ideal case (see below) the parameter ρ_M equals the superfluid density ρ_s, which at $T = 0$ and for the translationally invariant system equals the total mass density $\rho = mn$. The experiments on JJA show that ρ_M is either zero or is very small compared to ρ_s. There are several controversial exlpanations of the absence of the Magnus force.

(1) In Ref.[7,8] it is assumed that the phase ϕ_i of the condensate in the i-th island is canonically conjugated to the electric charge Q_i of the island. From this assumption it follows that the Magnus force is proportional to the offset charges on the superconducting islands, rather than to ρ. The effect of offset charges is negligible, in particular because the "real samples are usually characterized by random offset charges. As a result the Magnus force averages to approximately zero". The drawback of this approach is that because of the separate conservation law for the number of electrons, one should expect that the phase ϕ_i of the electron condensate is to be canonically conjugated to the number of the electrons N_i of the island, rather than to the charge Q_i of the island which is given by the difference in the numbers of electrons and protons.

(2) The more traditional point of view, that the Magnus force is proportional to the density of superconducting electrons on the islands averaged over distances large compared to the lattice constant of the array (see eg[9]), contradicts to the experiment. To match the experiment it was assumed in[10] that the force is proportional to the *local* superconducting density at the point where the vortex is situated. Since the vortex does not move through superconducting islands but through the junctions, the Magnus force on the vortex can be substantially reduced. This approach can be applied to the systems in which the Magnus force can be locally determined, for example if the order parameter changes smoothly on the distance of the core size. This is apparently not the case in the JJA.

(3) In Ref.[11] the absence of the Magnus force was ascribed to the nearly complete cancellation of the Magnus force by the spectral-flow force: $\rho_M = m(n - n_0)$, where n_0 deviates from the particle density n only due to small particle-hole asymmetry. Such cancellation is known for the bulk superfluids and superconductors in the so called hydrodynamic regime, where the momentum exchange between the electrons in the core and that in the heat bath is maximal. In this case the spectral flow along the low-energy levels of the bound states in the core of vortices almost completely cancels the Magnus force[12–15]. It was shown in[11] that similar spectral flow can take place in the case when the Josephson junctions are of the Superconductor–Normal-

metal–Superconductor (SNS) type. However this mechanism cannot be applied to the Josephson junctions of Superconductor–Insulator–Superconductor (SIS) type, where the low-energy levels are absent and the spectral flow is forbidden.

(4) Sonin[1] suggested that the effective Magnus force in the JJA is exactly zero due to exact symmetry of the motion equations which follows from the particle-hole symmetry. However if one continuously increases the transparency of the contact, ie the critical current through the Josephson contact, one finally reaches the limit of the bulk superconductivity, where the Magnus force is too well known to be nonzero, at least in the nonhydrodynamic regime. So one should assume that on the way from JJA to bulk superconductivity, there is a quantum Lifshitz transition from zero to nonzero value of the Magnus force. This does not seem very unreasonable, but one must take into account that the particle-hole symmetry is not exact in superconductors. There is always a small asymmetry of order Δ^2/E_F^2, where Δ is the superconducting gap and E_F is the Fermi energy. This asymmetry never disappears and thus the Magnus force is never exactly zero. Let us discuss how this asymmetry can enter the Magnus force in JJA.

The Magnus force in the systems without translational invariance still remains an open question, though this problem arises even in the bulk superconductor due to the band structure of the electrons in crystals[16]. The main problem is what electrons are involved in the construction of the transverse force acting on the vortex: are these the extra electrons due to the offset charge or all electrons; all superconducting electrons, or only the Cooper pairs concentrated in a small belt in momentum space with dimension of the gap Δ in the vicinity of the Fermi surface? It can be also the Andreev bound states in the vortex core or Bogoliubov quasiparticles in the normal component outside the core.

There is no unique answer to this question, since the result depends on the kinetics of the electrons on the background of the moving vortex. However there are some limiting cases in which the dissipation can be neglected and the answer can be guessed from the general principles before the detailed calculations. For example, if two conditions are fulfilled – (i) the system is translationally invariant and (ii) the transport is adiabatic – the Magnus force has its maximal *ideal* value determined at $T = 0$ by the total electron density: $\rho_M = mn$. This limit occurs eg. when there is a gap in the spectrum of the electrons even in the presence of vortex cores. In this case the results of calculations, made by groups with essentially different ideology, say[17] and[12], coincide.

If any of the two conditions are violated the life becomes more complicated. If the translational invariance (condition (i)) is absent, the Magnus force is to be reduced: some electrons are pinned by the crystal lattice and thus cannot contribute the transverse force. The pinned electrons are either those which belong to the completely occupied electronic bands[16], or the electrons localized on impurities.

The adiabatic condition (ii) is violated when the inverse relaxation time $1/\tau$ is comparable with the minigap – the interlevel distance of Andreev bound states in the vortex core – which is $\omega_0 \sim \Delta^2/E_F$. In this case one should either solve the transport equation or consider different limits – different classes with zero dissipation separated by the regions of the parameters, where the dissipation is finite. In the translationally invariant system these limits are determined by the value of the parameter $\omega_0\tau$: in the hydrodynamic regime, $\omega_0\tau \ll 1$, the Magnus force is very small $\rho_M \sim \rho\Delta^2/E_F^2$; in the collisionless regime, $\omega_0\tau \gg 1$, the *ideal* value of the Magnus force is restored, $\rho_M = \rho$. For the d-wave superconductors with anisotropic gap one has even two different parameters, coming from the semiclassical and the true (quantum) minigaps[14,15], and thus one has 3 different regimes.

In the hydrodynamic regime, $\omega_0\tau \ll 1$, the essential reduction of the Magnus force occurs due to the spectral flow of the electrons (momentum exchange between the core electrons and the heat bath). Formally this extreme spectral flow means the pinning of the core electrons by the heat bath or by the crystal lattice. That is why the effects of violation of conditions (i) and (ii) should be similar. Formally the electrons localized in the translationally noninvariant system have $\tau = 0$, which corresponds to the hydrodynamic regime and thus to the extreme spectral flow.

We argue that the origin of the negligibly small Magnus force in the SIS JJA results from the strong violation of the translation invariance, due to which almost all electrons are pinned within islands. It appears that the reduction of the Magnus force is effectively the same as in SNS JJA: the Magnus force is nonzero only due to the small asymmetry between particles and holes[11]. The origin of this coincidence is that both the spectral flow phenomenon in SNS JJA and the smallness of the Josephson coupling between the islands in SIS JJA lead to the similar effect of pinning of almost all the electrons: they do not follow the vortex dynamics and do not contribute the Magnus force.

Quantitatively the Magnus force is determined by the linear in $\dot{\phi}_i$ term of the effective action for the i-th island (see Eq.(97) of[1]):

$$\frac{1}{2}\tilde{N}_i\dot{\phi}_i \quad . \tag{2}$$

The Magnus force is proportional to $\tilde{N}$ according to Eq.(100) of Ref.[1]. The quantity $\tilde{N}$ can be calculated within the BCS theory applied to the superconductor in the i-th island. The calculation of the effective BCS action shows that the variable $\tilde{N}_i$, which is canonically conjugated to the phase ϕ_i, is not the total number N_i of electrons but is proportional the square of the gap amplitude (see eg. Ref.[18]). In a more general form the quantity $\tilde{N}_i$ can be written as[19]

$$\tilde{N}_i(\Delta) = \int_0^\Delta d\Delta' \frac{dN_i(\Delta')}{d\Delta'} = N_i(\Delta) - N_i(0) \quad , \qquad (3)$$

where Δ is again the gap amplitude and $N_i(\Delta)$ is the number of the electrons as a function of Δ at given chemical potential, if the Coulomb effects are neglected. The quantity $\tilde{N}_i$ differs from the actual number $N_i = N_i(\Delta)$ of the electrons in the superconducting island by the constant value $N_i(0)$. This $N_i(0) = N_i(\Delta = 0)$ is the number of electrons in the hypothetical normal state with $\Delta = 0$, which has the same chemical potential as the superconducting state, if the charging effect is neglected (see also Ref.[20]). The quantity $\tilde{N}_i$ is nonzero only due to the small asymmetry between the particles and holes and is of order $N_i\Delta^2/E_F^2$.

Since the parameter $N_i(0)$ is constant, it does not influence the classical dynamic equations for JJA, which thus remain symmetric in the sense discussed by Sonin. But this parameter $N_i(0)$ essentially influences the Magnus force, which contains the small factor $N_i - N_i(0)$ instead of N_i, ie $\rho_M \sim mn\Delta^2/E_F^2$.

As was noted in Ref.[18] the derivation of the "topological term" in the BCS action, which is linear in $\dot{\phi}_i$, does not depend on such details as the electronic mean free path and thus is the same in a clean and dirty limits. This is confirmed by the general form of the Eq.(3). That is why the Eq.(3) can be applied to many different limiting cases, though one should realize in which corner of the parameter space the system is. In our case the decription in terms of the phases ϕ_i of islands is to be valid, which implies that the critical current should be small enough. The detailed calculations are needed to understand how the Coulomb blockade can influence the magnitude of the topological term.

Thus the effective Lagrangian for JJA should contain two types of the "offset charges":

$$L = \frac{1}{2}\sum_{ij}(Q_i + Q_{1i})C_{ij}^{-1}(Q_j + Q_{1j})$$
$$-E_J \sum_{<ij>} \cos(\phi_i - \phi_j) \; + \; \frac{1}{2e}(Q_i + Q_{2i})\dot{\phi}_i \; . \qquad (4)$$

Here $Q_i = Q_{pi} - eN_i$ is the total electric charge of the island, where Q_{pi} is the charge of the positive ionic background; Q_{1i} is the conventional offset charge (see eg. Ref.[7]); while Q_{2i} is the "offset charge" coming from the particle-hole asymmetry. This charge $Q_{2i} = eN_i(0) - Q_{pi}$ is small due to the small value of the parameter Δ^2/E_F^2. The Magnus force is proportional to the average value of $< Q_i > + < Q_{2i} > = < Q_{2i} > - < Q_{1i} >$. While $< Q_{1i} >$ can be zero[18], the "offset charge" $< Q_{2i} >$ coming from the particle-hole asymmetry does not average to zero and should produce small but finite Magnus force.

In conclusion, contrary to the Sonin's arguments[1] the symmetry of the dynamic equations for the JJA does not forbid the so called topological term in the effective action for JJA even in the classical limit. This term leads to the finite effective Magnus force which is proportional to the parameter of the particle-hole asymmetry.

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